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Derivation of rupture area and stress-drop from body wave displacement spectra and the relative material strength in deep seismic zones

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The seismic moment and source area of an earthquake can be determined by fitting theoretical displacement amplitude spectra to observed ones. From these basic parameters the dislocation at the source and the stress-drop can be estimated. This method was tested in the case of four earthquakes for which the source parameters were known from observed surface ruptures. The uncertainty in the moment and area determinations was found to be approximately a factor of 2; for the displacement and stress-drop it was approximately a factor of 3 and 5 respectively. The application of spectral analysis of body waves to earthquakes in the deep seismic zone of Tonga-Kermadec indicate that stress-drop as well as apparent stress increase with depth and decrease again at great depth. This observation is interpreted as reflecting increasing material strength in the deep seismic zone near 450 km, with a reduction of strength at still greater depths. It is proposed that the temperature distribution in the downgoing slab of lithosphere causes this pattern.

Introduction

The spectra of all waves radiated from a seismic or explosive source will carry the stamp of the physics of the source process. Some of the physical source parameters can be determined from seismic waves under the following conditions: (1) the seismic waves must be recorded in an appropriate and wide enough frequency band so that the spectral characteristics can be defined reliably. (2) One must be able to correct the spectra for all the modifications which occurred along the path and in the receiver. (3) A reliable theory to relate spectral characteristics to source parameters is needed.

In this paper recent developments in the derivation of source parameters from P and S wave amplitude spectra will be summarized. The necessary steps of the analysis will be discussed, the reliability of the method will be estimated and a recent application of the method to a particular setting will be summarized.

In the far-field the P and S wave displacement amplitude spectra of earthquakes or explosions are characterized by a corner or peak frequency, f_0. This frequency is at the centre of the band width of interest, and in the time domain it corresponds to the predominant frequency recorded. f_0 has been related to the source dimensions by various authors (e.g. Kasahara 1957; Berckhemer & Jacob 1968; Brune 1970)

$$f_0 = Cv/r, \tag{1}$$

where v is the wave velocity near the source or the rupture propagation velocity depending on the source model used, and C is a constant of order 1 which also depends on the model used.

The shape of the displacement amplitude spectrum of a dislocation source is radically different on the two sides of f_0. For $f < f_0$ the spectral amplitudes are constant and can be related directly to the seismic moment M_0 (the strength of the source) by

$$M_0 = 4\pi\rho R v^3 \Omega_0 / \mathscr{R}_{\theta\phi} \quad \text{Keilis-Borok (1959).} \tag{2}$$

Ω_0 = long period spectral amplitude level of P or S wave; $\mathscr{R}_{\theta\phi}$ = radiation pattern of P or

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S wave; ρ = density near the source; R = hypocentral distance (in a layered, spherical Earth); v = P or S wave velocity. For $f < f_0$ the wavelengths λ are larger than the source dimensions and therefore the double couple point source approximation by Burridge & Knopoff (1964) is correct and leads to equation (2).

For $f \gg f_0$, $\lambda < r$ in Brune's model and the spectral amplitude decreases rapidly, in the order of f^{-2}. This part of the spectrum is most sensitive to the short time behaviour of the source. Most of the energy is associated with frequencies $f \geqslant f_0$ since the observed spectra tend to decrease with f^{-1} to f^{-2} near f_0 and the energy in an elastic wave is

$$E \approx \rho \int [\Omega(\omega)\ \omega]^2\, \mathrm{d}\omega, \tag{3}$$

where $\omega = 2\pi f$ and Ω is the spectral amplitude.

For the derivation of source parameters from body waves we proceed as follows: the radiation pattern is obtained from the first motions of P waves and the polarization of S waves. Then the amplitude spectrum of a P or S wave at a suitable distance is obtained in as wide a frequency band as possible. This station spectrum is then corrected for all the modifying effects of the path and the receiver. The radiation pattern correction is applied and thus we obtain the source spectrum. If several stations are available for analysis a more reliable estimate of the average source spectrum is obtained by taking the r.m.s. average spectrum.

From this we first obtain the observed long period level Ω_0 and then we compute M_0 from (2). Then we calculate r from f_0 using (1) and we obtain an estimate of the total energy in P or S waves by integrating first the energy in the frequency band of the spectrum and then integrating in space the energy radiated through a sphere around the source, considering the known radiation pattern (Wu 1966).

From these three basic parameters M_0, E_s and r which are directly observed we calculate additional parameters of interest using the definitions of M_0 and E_s in terms of source parameters.

$$M_0 = \mu D A \quad \text{(Aki 1966)}, \tag{4}$$

$$E_s = \eta E = \eta\, \tfrac{1}{2}(\sigma_1 + \sigma_2)\, DA = \eta\bar{\sigma} DA, \tag{5}$$

where E_s is a fraction η of the elastic work E done over the fault area A by a change of shear stress across A from σ_1 to σ_2. The average displacement over A was D.

Using the approximation

$$A = \pi r^2.$$

which is good for medium sized and deep earthquakes we may evaluate D using (4) the observed M_0 and r.

From the same basic parameters one can compute the stress-drop $\Delta\sigma = \sigma_1 - \sigma_2$

$$\Delta\sigma = \tfrac{7}{16} M_0 / r^3 \quad \text{(Brune 1970)}. \tag{6}$$

Next we can estimate the apparent stress $\eta\bar{\sigma}$, the product of efficiency η and the average stress by taking the ratio E_s/M_0 and using (4) (5). However we cannot determine the initial or the final stress from seismic waves without an additional observation like *in situ* stress at the source, etc. (Orowan 1960).

The determination of the parameters r, A, D, $\Delta\sigma$ all depend on the constant C in equation (1). Hanks & Wyss (1972) have tested a number of theories and found that Brune's (1970) model fits field observations best. This test and the estimated accuracy of the method are summarized in a later section.

Two additional parameters can be estimated from the amplitude spectra of body waves under ideal circumstances: the rupture velocity v_r and the fractional stress-drop ϵ (Brune 1970). The effect of a propagating source is to focus energy in the direction of propagation (see, for example, Ben-Menahem 1961) which means that f_0 and the $\eta\bar{\sigma}$ observation in this direction are increased. For medium-sized earthquakes this effect seems to be very small if observable at all (Berckhemer & Jacob 1968; Hanks & Wyss 1972). According to Brune's (1970) model it should be possible to estimate the fractional stress drop from the amplitude spectra. However, this determination depends on the exact knowledge of the high frequency amplitudes. These, however, are not very well known because of the uncertainties associated with the attenuation correction as we shall see below.

Time domain or frequency domain analysis

The process of source parameter derivation described above can of course be done in the time domain. In this case the seismic waves produced at teleseismic distances in a realistic Earth model are computed for a given seismic source. This result is put theoretically through the recording instrument and thus theoretical seismic recordings are compared to observed ones. From the best fit, source parameters are then determined (Mikumo 1971, 1972).

The spectral approach has two important advantages. First, the observer can inspect the source spectrum directly whereas in the time domain the fit of a severely modified pulse is considered. Secondly, because the frequency bands of most seismographs are narrow, it is very useful to produce a combined broad band spectrum from the analysis of two or more instruments. Without the joint use of short and long period instruments of the World Wide Standard Seismograph Network (W.W.S.S.N.) the source dimensions of most shocks with $m_b \leqslant 6$ cannot be estimated with confidence, since the corner frequency for these events is near the high resolution limit of the long period instruments.

Limitations imposed by window length and instrument characteristics

Waves of different types and waves that take different paths through the earth should not be mixed in the spectral analysis, because they are differently related to the source parameters. The window for Fourier analysis must be chosen so that only P or S is included. Phases like pP, PcP, sS, ScS thus limit the maximum window length. On the other hand, it is desirable to take the longest possible window because the spectral amplitudes of periods longer than the window length cannot be estimated. The optimum window starts just before the analysed phase (P) and stops just before the next phase (e.g. pP). If only periods smaller than the window length are considered the problems discussed by Linde & Sacks (1971) do not occur.

Other limitations to the resolution at the long as well as short end of the spectrum are imposed by the instrument characteristics. The response of any seismograph is limited to a certain band width and most instruments are peaked at an optimal period for registration. On either side they drop sharply to avoid the magnification of noise. Errors introduced by the analysis will be magnified through the instrument response correction at periods of low magnification. It is therefore only safe to trust results of waves with periods near the peak of the instrument magnification and waves which are predominant on the seismic record.

In addition, at high frequencies a cut-off of information occurs below the period of highest sampling rate. Most long-period W.W.S.S.N. instruments record at 1 min on 1.5 cm paper length. The highest sampling rate of 0.5 mm (about the width of the trace) implies that the resolution is only above approximately 3 s period. For these reasons Wyss *et al.* (1970) have started to patch together broad earthquake spectra from instruments with different peak amplifications.

On the short period W.W.S.S.N. records a small problem arises with waves which have travelled other than the direct path. These waves arrive a short time after P. They are easily excluded by taking a time window in the order of 10 s which is long enough to define the short period of the spectra of most earthquakes since the source duration is typically shorter or equal to 10 s.

In summary it can be said that the combination of the long- and short-period W.W.S.S.N. instruments covers reliably the band $100 \geqslant T \geqslant 0.5$ s. At the short period end analysis errors are far smaller than the uncertainties associated with the attenuation correction, and at the long period end the reliability of the spectrum can easily be checked by surface wave analysis (see below).

Corrections for amplitude modifications along the path

In order to derive the source parameters we must correct the observed spectrum back to the source spectrum accounting for the effects alternating the amplitudes along the path.

Geometrical spreading in a layered spherical Earth can be calculated straightforwardly. Upper mantle discontinuities cause large amplitude fluctuations at distances shorter than 40° (see, for example, Julian & Anderson 1968). Because these amplitude changes depend critically on the upper mantle structure, it is safer to limit spectral amplitude analysis to distances larger than 40°. For such distances the rays depart from the source at angles mostly steeper than 30° from the vertical, spending only a short time in the problematic upper mantle.

Because PcP and ScS merge with P and S at large distances, and we wish to exclude the reflected phases, the best distance interval for the analysis is 40 to 70°.

Inhomogeneities near the source are particularly strong for foci occurring in deep seismic zones. Various authors have shown that a slab of high velocity exists there (e.g. Mitronovas & Isacks 1971) with high Q values next to a zone of extremely low Q values at the concave side of the island arc (Niazi 1971; Barazangi & Isacks 1971). These inhomogeneities give rise to a defocusing effect in the direction of the dipping slab (see, for example, Toksöz, Minear & Julian 1971). It is therefore important for amplitude studies to concentrate on rays which depart steeply from the source, leaving the slab almost immediately. It is of particular importance to avoid rays which travelled above the slab on the concave side because of the large and possible irregular attenuation there. These dangers are of course greatest for shallow earthquakes in island arc systems. However, they are all avoided when only distances exceeding 40 to 50° are considered.

Inhomogeneities below the stations can be corrected in detail if the local structure is well enough known. The basic effect of the crust and free surface on the amplitude spectrum is an increase due to the lower density in the crust and the reflexion at the free surface. Ben-Menahem, Smith & Teng (1965) have studied this problem in detail showing how interference gaps and amplitudes of the spectra changed as a function of crustal structure and angle of incidence.

It would be very desirable to correct every spectrum for the crustal structure beneath each station, however for most stations the structure is not well known and for two reasons the detailed correction is not really necessary. First of all we need not explain every trough and peak in the spectrum since our analysis will only depend on the overall spectral shape. Secondly, it will be necessary for other reasons to consider the average spectrum of several stations and it will be enough to correct for the average amplification. This correction can be approximated by dividing the amplitudes at all frequencies by a factor of 2.5.

Attenuation by inelastic effects is the least known factor. The decrease in amplitude along a ray-path may be approximated by

$$A(f, t) = A_0 \exp(-\pi t f/Q) = A_0 \exp(-\pi X f/Qv), \tag{7}$$

where A_0 is the initial amplitude, t is the travel time, f is the frequency, Q is the attenuation coefficient, X is the distance and v is the wave velocity.

Since the attenuation is frequency dependent, the shape of the spectrum can be changed if high frequencies are considered at too large distances. For the teleseismic analysis this is no problem because the available instruments allow analysis of $m_b \approx 5.5$ and larger events only. For this size events the corner frequency occurs at approximately 0.1 Hz. Only at frequencies higher than this will the attenuation effect start to exceed 20% for both P and S (Julian & Anderson 1968). For events well recorded by the W.W.S.S.N. one therefore will be able to estimate f_0 correctly but not the total energy which depends strongly on the high frequency amplitudes.

For local recordings the Q correction may be more of a problem. For a small earthquake of $M_L = 1$ the corner may occur at 10 Hz, and the surface layers may have a Q in the vicinity of 200. One can see from (7) that the attenuation at distances exceeding 10 km will be strong and if Q is not known in detail one may no longer consider the corner frequency to be well determined.

The radiation pattern correction depends on the geometrical orientation of the fault planes, the azimuth and distance to the station. One might argue for an average correction when many stations with different azimuth are used; however, we prefer to determine the exact factor for every station because near nodes the decrease in amplitude may become very strong, i.e. the departure from the average could theoretically be very large. Also the radiation pattern is easily determined from first motion studies (Ben-Menahem *et al.* 1965). In addition, we wish to know which stations are located in the direction of the fault planes because we may be able to observe the effects of rupture propagation.

Observational check on theoretical spectra

As there are a number of theoretical source spectra proposed, and since they differ in the relationship of corner frequency to source dimensions it is important to check by direct observation which theory is best calibrated. A full check can be made for earthquakes with surface rupture plus well studied aftershock zones. From the length of the surface rupture and aftershock zone the area of rupture can be estimated to within 20%. Then one has to assume that the surface displacement equals the average displacement on the fault plane. This assumption may be in error by as much as a factor of 3. From displacement and dimensions one then computes the moment and the stress-drop using equations (1) and (2). The seismic moment thus obtained from the field observations can then be compared to the moment obtained from surface waves, P waves and S waves. The source dimensions obtained in the field can be compared to those estimated from f_0 of P and S waves. D as well as $\Delta\sigma$ can be compared. Hanks & Wyss (1972) and Wyss & Hanks (1972) have performed this check on three strike-slips and one thrust earthquake. They found that the theory by Brune (1970) gives excellent agreement for the strike-slip events. The agreement in the case of the thrust event is good if the surface displacement along the rupture is considered. However, a recent analysis of geodetic observations (Canitez & Toksöz 1972) suggests that displacement at depth was larger, which would make the agreement poor. This raises a question which still has to be resolved.

It is felt that it is important to perform this check on individual events, i.e. to test the relation $r = r(f_0)$ in the case of actually observed spectra rather than base the relation on the average magnitude fault length relation, since there is overwhelming evidence that order of magnitude differences exist for a given moment or magnitude (Wyss & Brune 1971; Thatcher 1972; Tsujiura 1969).

A disadvantage of the test based on an earthquake with surface rupture is that pP cannot be excluded. That is, some sort of surface reflexion near the source is included in the analysis. This, however, seems not to be a serious problem since the field, surface wave and body wave moments are all in excellent agreement with each other.

Brune (1970) proposed his model for S waves. Hanks & Wyss (1972) also tested the extension of equation (3) to P waves by substituting the P wave velocity α for β the S wave velocity and found that it was successful. This extension of the model is based on the idea that the critical wavelength $\lambda_0(f_0)$ for P and S waves has to be the same since for $\lambda > \lambda_0$ the point source model is applicable, whereas for $\lambda < \lambda_0$ the wavelengths are shorter than the source dimensions (Wyss *et al.* 1970). It can be said in summary that Hanks & Wyss (1972) found that using S and P wave spectra and Brune's (1970) theory, the seismic moment can be estimated from P and S waves to within a factor of 2; the area to within a factor of 2; the displacement and stress drop to within a factor of 3 to 5. These error estimates are only guesses of what seems to be plausible considering the variations in the data for the four earthquakes analysed. Because of the poorly known attenuation correction, the energy estimate was considered by Hanks & Wyss (1972) to be uncertain by a factor of 10.

Since the check on the theory was performed with good results we can now proceed to determine source parameters of earthquakes in different tectonic environments. We will here make the assumption that the check on the theory performed for shallow events is also valid for deep events. It is felt that this assumption is justified because the fault plane solutions of deep earthquakes indicate the same double-couple source mechanism as for shallow events. In addition, it gives confidence to know that the surface wave and body wave moments for deep events agree with each other.

Application of the calibrated method to an island arc

The physical source mechanism of deep earthquakes is still undetermined. Since there are a number of hypotheses proposed it is hoped that the determination of stress-drop and apparent stress as a function of depth may help to rule out one or the other hypothesis.

The first extensive study of spectral properties as a function of depth was done by Kasahara (1957). On the basis of the calibration check (Hanks & Wyss 1972) Brune's (1970) model is preferred to Kasahara's (1957) explosion type source mechanism. On a relative basis, however, Kasahara's conclusion is still valid: he found that in the seismic zones of Japan and Tonga the source dimensions of a given magnitude did not change significantly with depth.

Wyss (1970) found that the apparent stress in the South America seismic zone was an order of magnitude higher at intermediate depth than for shallow and deep earthquakes, and Tsujiura (1969; personal communication) obtained similar qualitative data for the Aleutians and Tonga regions. Several other authors have determined stress-drops for a number of deep earthquakes (Berkhemer & Jacob 1968; Mikumo 1970, 1971; Bollinger 1969; Fukao 1971).

The first extensive study using the calibrated theory by Brune (1970) was done for 53 earthquakes in the Tonga–Kermadec zone by Molnar & Wyss (1972) and Wyss & Molnar (1972). These authors used five to six stations for each earthquake to obtain average P and S wave spectra, from which they determined stress-drops and apparent stress. Their results are reproduced as a function of depth in figure 1.

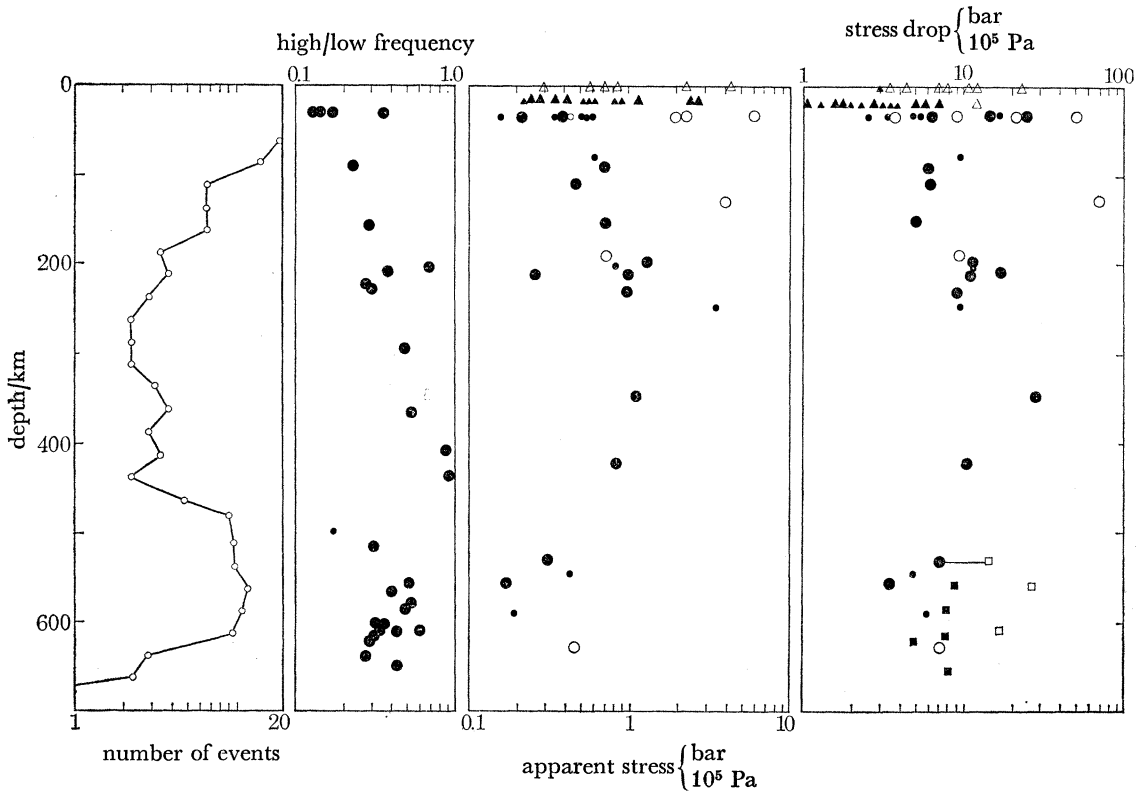

FIGURE 1. Source parameters as a function of depth in the Tonga–Kermadec arc. Number of earthquakes from Sykes (1966), amplitude ratio from Tsujiura (personal communication), stress-drop and apparent stress from Wyss & Molnar (1972). All source parameters show a discontinuity near 450 km depth.

Molnar & Wyss (1972) found that shallow earthquakes on the plate boundary had lower stress-drops than earthquakes within the plates. Because non-shallow earthquakes occur within the plates (Isacks *et al.* 1968; Isacks & Molnar 1969; Davies & Brune 1970) Wyss & Molnar (1972) argued that the source parameters of these events must be compared with the shallow earthquakes within plates. On this basis of comparison they concluded that stress-drop and apparent stress are approximately constant to a depth of 450 km, at which depth a decrease in these two parameters occurs. Their results are compared in figure 1 to M. Tsujiura's (personal communication), who studied the amplitude ratio of waves with a frequency of 3.2 Hz to waves with 1.6 Hz. Tsujiura's results support the conclusion that a sharp decrease in high-frequency content (or apparent stress) occurs at approximately 450 km depth.

Another parameter shows a discontinuity at the same depth; Sykes (1966) showed that the number of earthquakes increases sharply at 450 km depth. His results are reproduced in figure 1 without the shallow earthquakes because most of those are located on the boundary between plates and not within them. The rate of activity at the boundary cannot be compared with the rate within plates.

In summary, it is evident from figure 1 that none of the measured source parameters in the Tonga–Kermadec arc change very much with depth. This may be a characteristic of continuous deep seismic zones. The facts that source dimensions do hardly change with depth (Wyss & Molnar 1972), and the orientation of fault planes does not change (Isacks *et al.* 1968) suggest that the same fault area is ruptured again and again as it travels down into the mantle. In addition, the dislocations during ruptures do not change appreciably, producing constant stress-drops (figure 1) down to 450 km. This indicates a remarkable independence of the stress-drop from the confining pressure, whereas in the laboratory the stress-drop increases with confining pressure.

It is hoped that the weakening of the slab at 450 km depth may furnish a clue as to whether a dehydration mechanism (Raleigh & Paterson 1965), a shear melting mechanism (Griggs & Baker 1969) or a phase change (Anderson 1967; Kanamori 1970; Anderson & Demarest 1971) is causing earthquakes at great depth.

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